

# SmallClassNr

## Library of finite groups with small class number

1.4.1

13 September 2025

**Sam Tertooy**

**Sam Tertooy**

Email: [sam.tertooy@kuleuven.be](mailto:sam.tertooy@kuleuven.be)

Homepage: <https://stertooy.github.io/>

Address: Wiskunde

KU Leuven, Kulak Kortrijk Campus

Etienne Sabbelaan 53

8500 Kortrijk

Belgium

## **Abstract**

The `SmallClassNr` package provides access to finite groups with small class number. Currently, it contains the finite groups of class number at most 14.

## **Copyright**

© 2022–2025 Sam Tertooy

The `SmallClassNr` package is free software, it may be redistributed and/or modified under the terms and conditions of the [GNU Public License Version 2](#) or (at your option) any later version.

## **Acknowledgements**

This documentation was created using the `GAPDoc` and `AutoDoc` packages.

# Contents

<b>1</b>	<b>The SmallClassNr package</b>	<b>4</b>
1.1	Installation . . . . .	4
1.2	Loading . . . . .	4
1.3	Citing . . . . .	4
1.4	Support . . . . .	5
<b>2</b>	<b>Classification</b>	<b>6</b>
<b>3</b>	<b>The Small Class Number Library</b>	<b>7</b>
3.1	Functions . . . . .	7
	<b>References</b>	<b>10</b>
	<b>Index</b>	<b>11</b>

# Chapter 1

## The SmallClassNr package

This is the manual for the GAP 4 package SmallClassNr version 1.4.1, developed by Sam Tertooy.

### 1.1 Installation

You can download SmallClassNr as a .tar.gz archive [here](#). After extracting, you should place it in a suitable pkg folder. For example, on a Debian-based Linux distribution (e.g. Ubuntu, Mint), you can place it in \$HOME/.gap/pkg (recommended) which makes it available for just yourself, or in the GAP installation directory (gap-X.Y.Z/pkg) which makes it available for all users.

You can use the following command to efficiently install the package for yourself:

```
Command
wget -q0- https://[...].tar.gz | tar xzf - --one-top-level=$HOME/.gap/pkg
```

If the PackageManager package is installed and loaded, you can install SmallClassNr from within a GAP session using InstallPackage (**PackageManager: InstallPackage**).

```
Example
gap> InstallPackage( "https://[...].tar.gz" );
...
true
```

### 1.2 Loading

Once installed, loading SmallClassNr can be done by using LoadPackage (**Reference: LoadPackage**).

```
Example
gap> LoadPackage( "SmallClassNr" );
...
true
```

### 1.3 Citing

If you use the SmallClassNr package in your research, we would love to hear about your work via an email to the address [sam.tertooy@kuleuven.be](mailto:sam.tertooy@kuleuven.be). If you have used the SmallClassNr package in the preparation of a paper and wish to refer to it, please cite it as described below.

In Bib<sub>T</sub><sub>E</sub>X:

Bib<sub>T</sub><sub>E</sub>X

```
@misc{SCN1.4.1,
  author = {Tertooy, Sam},
  title = {{SmallClassNr,
    Library of finite groups with small class number,
    Version 1.4.1}},
  note = {GAP package},
  year = {2025},
  howpublished = {\url{https://stertooy.github.io/SmallClassNr}}
}
```

In Bib<sub>L</sub><sub>A</sub><sub>T</sub><sub>E</sub>X:

Bib<sub>L</sub><sub>A</sub><sub>T</sub><sub>E</sub>X

```
@software{SCN1.4.1,
  author = {Tertooy, Sam},
  title = {SmallClassNr},
  subtitle = {Library of finite groups with small class number},
  version = {1.4.1},
  note = {GAP package},
  year = {2025},
  url = {https://stertooy.github.io/SmallClassNr}
}
```

## 1.4 Support

If you encounter any problems, please submit them to the [issue tracker](#). If you have any questions on the usage or functionality of SmallClassNr, you may contact me via email at [sam.tertooy@kuleuven.be](mailto:sam.tertooy@kuleuven.be).

## Chapter 2

# Classification

The *class number*  $k(G)$  of a group  $G$  is the number of conjugacy classes of  $G$ . In 1903, Landau proved in [Lan03] that for every  $n \in \mathbb{N}$ , there are only finitely many finite groups with exactly  $n$  conjugacy classes. The `SmallClassNr` package provides access to the finite groups with class number at most 14.

These groups were classified in the following papers:

- $k(G) \leq 5$ , by Miller in [Mil11] and independently by Burnside in [Bur11]
- $k(G) = 6, 7$ , by Poland in [Pol68]
- $k(G) = 8$ , by Kosvintsev in [Kos74]
- $k(G) = 9$ , by Odincov and Starostin in [OS76]
- $k(G) = 10, 11$ , by Vera López and Vera López in [VLVL85] (1)
- $k(G) = 12$ , by Vera López and Vera López in [VLVL86] (2)
- $k(G) = 13, 14$ , by Vera López and Sangroniz in [VLS07]

(1) In [VLVL85], three distinct groups of the form  $(C_5 \times C_5) \rtimes C_4$  order 100 with class number 10 are given. However, only two such groups exist, being the ones with `IdClassNr` equal to [10, 25] and [10, 26].

(2) In [VLVL86], only 48 groups with class number 12 are listed. The three missing groups are provided in the appendix of [VLS07]. These are the groups with `IdClassNr` equal to [12, 13], [12, 16] and [12, 39].

## Chapter 3

# The Small Class Number Library

### 3.1 Functions

#### 3.1.1 SmallClassNrGroup

▷ `SmallClassNrGroup(k, i)` (function)

**Returns:** the  $i$ -th finite group of class number  $k$  in the library.

Alternatively, the pair  $[k, i]$  can be given as a single argument  $id$ . If the group is solvable, it is given as a `PcGroup` whose `Pcgs` is a `SpecialPcgs`. If the group is not solvable, it will be given as a permutation group of minimal permutation degree and with a minimal generating set.

Example

```
gap> G := SmallClassNrGroup( 6, 4 );
<pc group of size 18 with 3 generators>
gap> NrConjugacyClasses( G );
6
gap> IsDihedralGroup( G );
true
```

#### 3.1.2 SmallClassNrGroupsAvailable

▷ `SmallClassNrGroupsAvailable(k)` (function)

**Returns:** `true` if the finite groups of class number  $k$  are available in the library, and `false` otherwise.

Example

```
gap> SmallClassNrGroupsAvailable( 14 );
true
gap> SmallClassNrGroupsAvailable( 15 );
false
```

#### 3.1.3 AllSmallClassNrGroups

▷ `AllSmallClassNrGroups(arg)` (function)

**Returns:** all finite groups with certain properties as specified by  $arg$ .

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

## Example

```

gap> L1 := AllSmallClassNrGroups( [3..5], IsNilpotent );
[ <pc group of size 3 with 1 generator>,
  <pc group of size 4 with 2 generators>,
  <pc group of size 4 with 2 generators>,
  <pc group of size 5 with 1 generator>,
  <pc group of size 8 with 3 generators>,
  <pc group of size 8 with 3 generators> ]
gap> List( L1, NrConjugacyClasses );
[ 3, 4, 4, 5, 5, 5 ]
gap> L2 := AllSmallClassNrGroups( IsSolvable, true, NrConjugacyClasses, 6 );
[ <pc group of size 6 with 2 generators>,
  <pc group of size 12 with 3 generators>,
  <pc group of size 12 with 3 generators>,
  <pc group of size 18 with 3 generators>,
  <pc group of size 18 with 3 generators>,
  <pc group of size 36 with 4 generators>,
  <pc group of size 72 with 5 generators> ]
gap> ForAll( L2, G -> IsSolvable( G ) and NrConjugacyClasses( G ) = 6 );
true

```

### 3.1.4 OneSmallClassNrGroup

▷ OneSmallClassNrGroup(arg)

(function)

**Returns:** one finite group with certain properties as specified by *arg*.

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

## Example

```

gap> H := OneSmallClassNrGroup( 6, IsAbelian );
<pc group of size 6 with 2 generators>
gap> IsCyclic( H );
true
gap> K := OneSmallClassNrGroup( 10, IsSolvable, true, IsNilpotent, false );
<pc group of size 28 with 3 generators>
gap> NrConjugacyClasses( K ) = 10 and IsSolvable( K ) and not IsNilpotent( K );
true

```

### 3.1.5 NrSmallClassNrGroups

▷ NrSmallClassNrGroups(arg)

(function)

**Returns:** the number of finite groups with certain properties as specified by *arg*.

The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be `NrConjugacyClasses`. Missing functions will be interpreted as `NrConjugacyClasses`, missing values as `true`.

## Example

```

gap> NrSmallClassNrGroups( 14 );
93
gap> NrSmallClassNrGroups( [3..5], IsNilpotentGroup );
6

```



```
gap> NrSmallClassNrGroups( IsSolvable, true, NrConjugacyClasses, 6 );
7
```

### 3.1.6 IteratorSmallClassNrGroups

▷ IteratorSmallClassNrGroups(*arg*) (function)

**Returns:** an iterator that iterates over the finite groups with properties as specified by *arg*. The arguments must come in pairs consisting of a function and a value (or list of possible values). At least one of the functions must be NrConjugacyClasses. Missing functions will be interpreted as NrConjugacyClasses, missing values as true.

Example

```
gap> iter := IteratorSmallClassNrGroups( IsSolvable, false, 11 );
<iterator>
gap> for G in iter do Print( Size( G ), "\n" ); od;
336
720
720
1344
1344
1512
2448
29120
```

### 3.1.7 IdClassNr

▷ IdClassNr(*G*) (attribute)

**Returns:** the SmallClassNr ID of *G*, i.e. a pair [ *k*, *i* ] such that *G* is isomorphic to SmallClassNrGroup( *k*, *i* ).

Example

```
gap> IdClassNr( AlternatingGroup( 5 ) );
[ 5, 8 ]
gap> A := SmallClassNrGroup( 5, 8 );
Group([ (1,2,3), (1,4,5) ])
gap> IsAlternatingGroup( A );
true
```

# References

- [Bur11] William Burnside. *Theory of groups of finite order*. The University Press, second edition, 1911. [6](#)
- [Kos74] L. F. Kosvintsev. Over the theory of groups with properties given over the centralizers of involutions. *Sverdlovsk (Ural.), Summary thesis Doct*, 1974. [6](#)
- [Lan03] Edmund Landau. Über die Klassenzahl der binären quadratischen Formen von negativer Discriminante. *Math Ann*, 56(4):671–676, 1903. [6](#)
- [Mil11] George Abram Miller. Groups involving only a small number of sets of conjugate operators. *Arch. der Math. u. Phys.*, 17:199–204, 1911. [6](#)
- [OS76] V. A. Odincov and A. I. Starostin. Finite groups with 9 classes of conjugate elements. *Ural. Gos. Univ. Mat. Zap*, 10:114–134, 1976. [6](#)
- [Pol68] John Poland. Finite Groups with a given Number of Conjugate Classes. *Canadian J Math*, 20:456–464, 1968. [6](#)
- [VLS07] Antonio Vera López and Josu Sangroniz. The finite groups with thirteen and fourteen conjugacy classes. *Math Nachr*, 280(5-6):676–694, 2007. [6](#)
- [VLVL85] Antonio Vera López and Juan Vera López. Classification of finite groups according to the number of conjugacy classes. *Isr J Math*, 51(4):305–338, 1985. [6](#)
- [VLVL86] Antonio Vera López and Juan Vera López. Classification of finite groups according to the number of conjugacy classes II. *Isr J Math*, 56(2):188–221, 1986. [6](#)

# Index

AllSmallClassNrGroups, [7](#)

IdClassNr, [9](#)

IteratorSmallClassNrGroups, [9](#)

NrSmallClassNrGroups, [8](#)

OneSmallClassNrGroup, [8](#)

SmallClassNrGroup, [7](#)

SmallClassNrGroupsAvailable, [7](#)